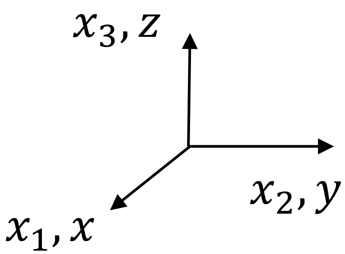
Reynolds-averaged simulation of Langmuir circulation in shallow water

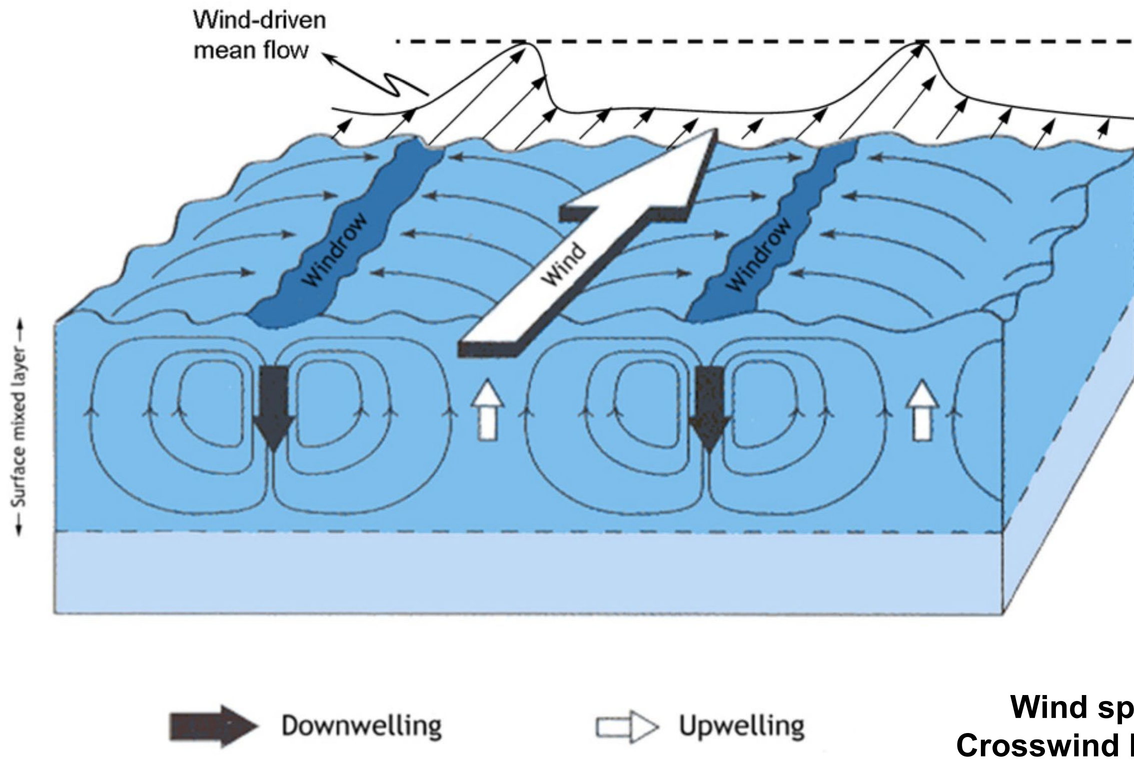
Andrés E. Tejada-Martínez

Civil and Environmental Engineering

University of South Florida, Tampa, FL

Observed structure of Langmuir circulations (LC)





Wind speed > 3 m/s
Crosswind length: 2-300 m
Downwind length: 200-3000 m

- **Langmuir cells historically have been observed in the mixed layer over deep water**
- **In shallow coastal regions (10-20 m in depth) the cells have been observed engulfing the entire water column during the passage of storms (Ann Gargett and collaborators)**

Surface manifestations of LC

- Langmuir turbulence is characterized by a range of windrow (or Langmuir) scales as can be seen during strong wind and wave forcing in a storm

- Under calmer conditions, the turbulent nature of this process can still be seen through the irregularities of the largest (most visible) of the scales

Windrows during a storm

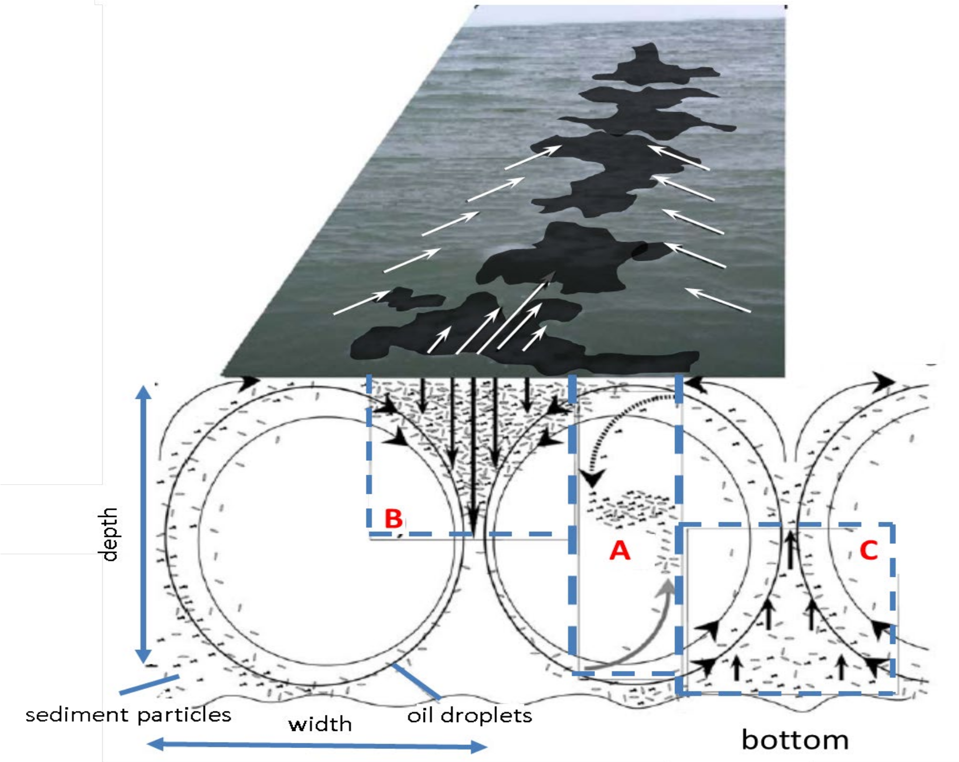
Windrows during calmer conditions

Observed structure of Langmuir circulations (LC)

Full-depth Langmuir cells can induce sediment resuspension

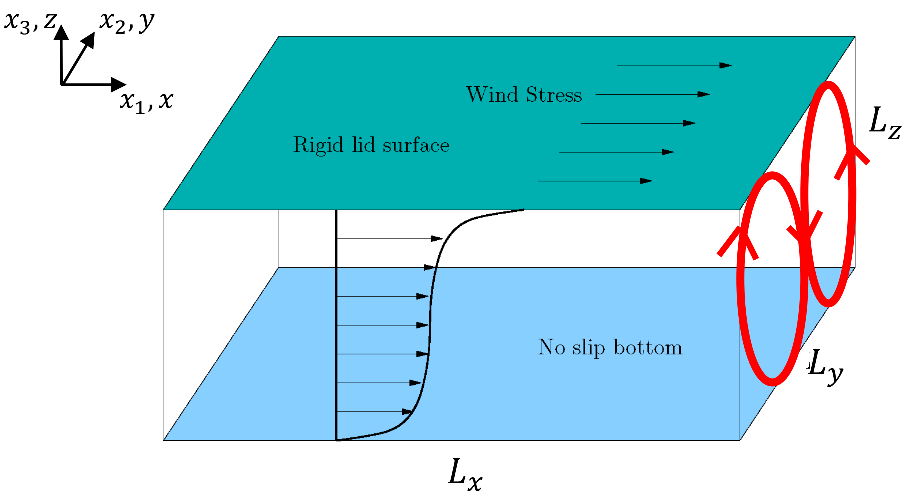
In the presence of an oil spills the cells can also lead to entrainment of oil droplets

Mixing of sediments and oil droplets can lead to the formation of oil-particle aggregates (OPA)



LES problem configuration

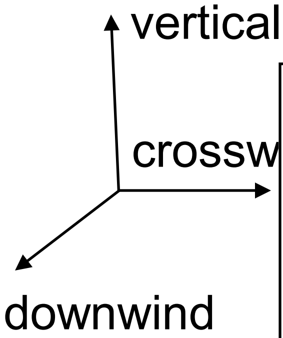
- **Large-eddy simulation (LES) performed within the following problem configuration:**



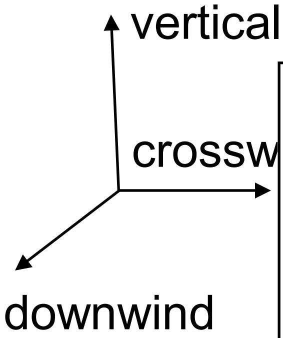
- Boundary conditions
  - Top is a wind-sheared rid lid
  - No-slip bottom
  - Periodicity in x and y

- For this configuration, formulation resolves
  - wind-driven mean current
  - largest LC (i.e. full-depth LC)

- Following the Gargett et al. field measurements of full-depth LC:
  - $L_z = 15\ m$
  - $L_y = 62.8\ m$ to resolve a pair of full-depth Langmuir cells

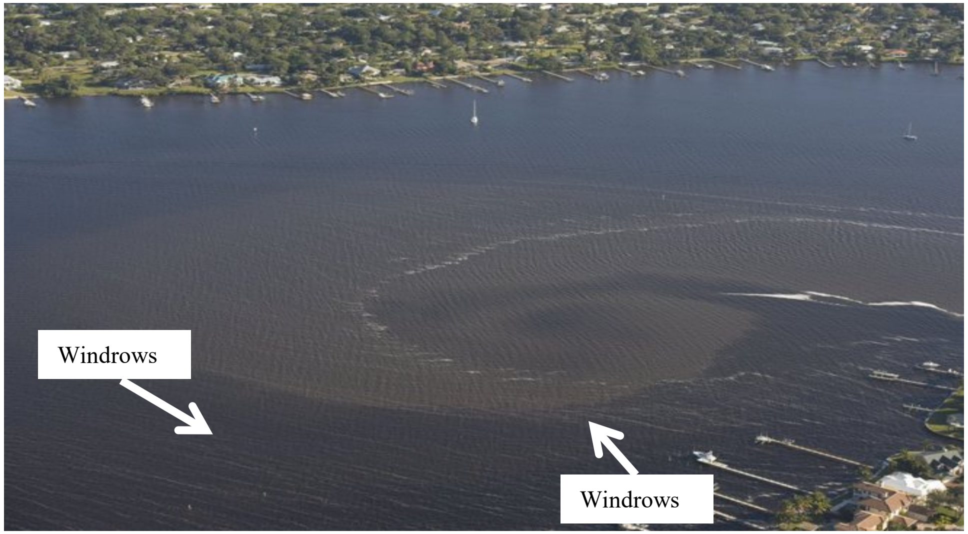


LC structur

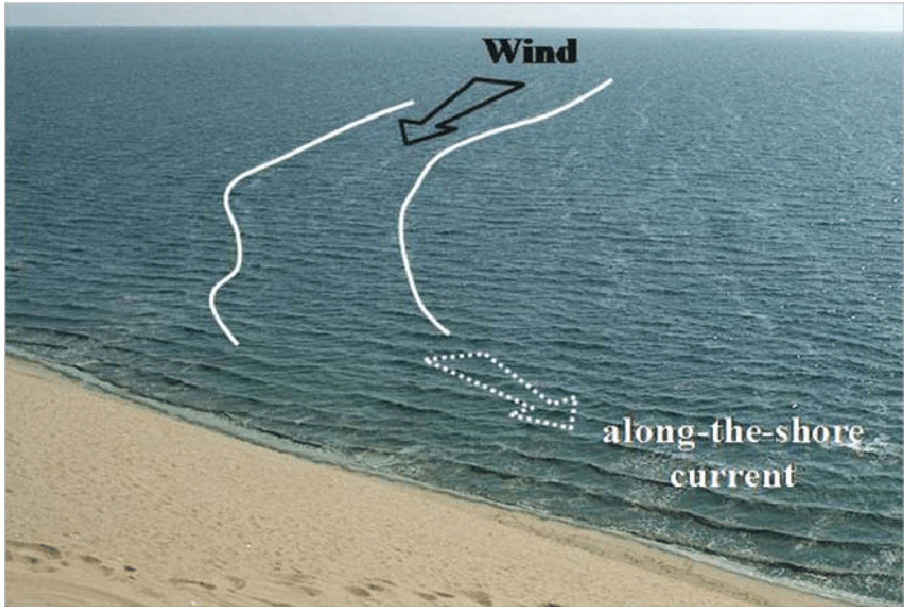


LC structur under oscillating crosswind body force

Surface manifestations of LC



Windrows interacting with a plume or eddy of murky water discharged from a river



Windrows interacting with an along-shore current

Goal

- Develop a Reynolds-averaged methodology that simulates (resolves) the largest of the Langmuir cells

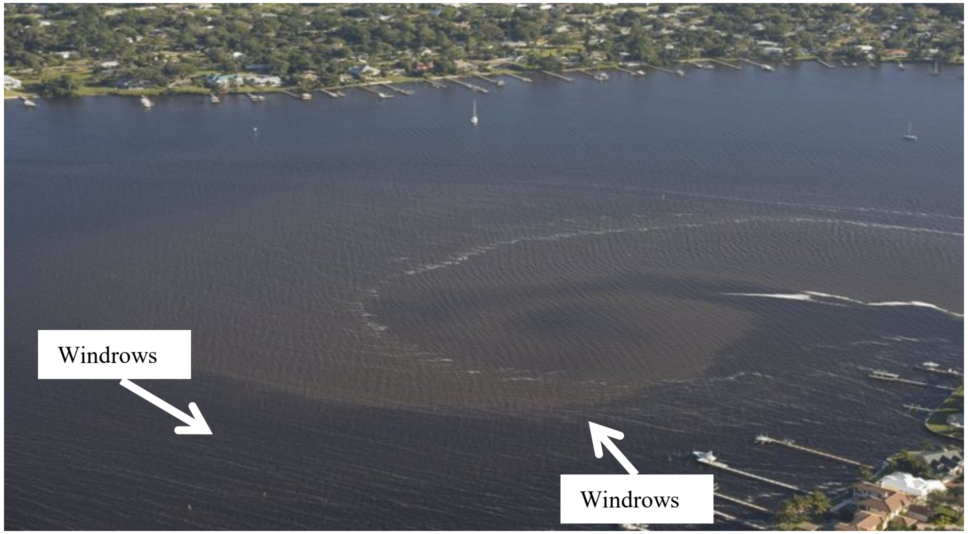
- In the case of the shallow coastal ocean or lakes and rivers, these cells would be the full-depth Langmuir cells observed by Gargett and collaborators

- The methodology should be equipped with a turbulence model accounting for the unresolved Langmuir scales (smaller than say the full-depth cells)

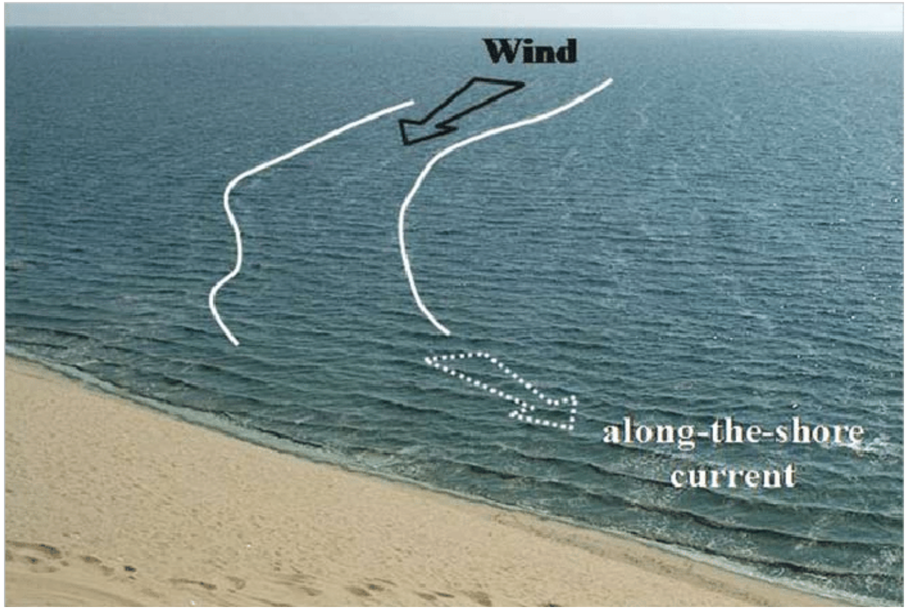
- This methodology would allow for estuarine and coastal circulation models to resolve the largest of the Langmuir cells.

- Ultimately, this methodology could prove useful for understanding how the largest of the Langmuir cells interact with shores, bottom topography and larger coherent features such plumes, lateral eddies and fronts

Surface manifestations of LC



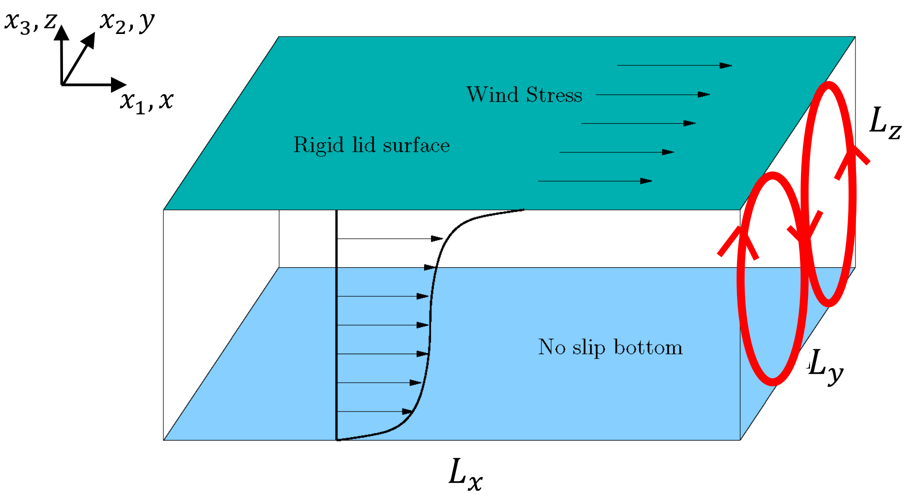
Windrows interacting with a plume or eddy of murky water discharged from a river



Windrows interacting with an along-shore current

Problem configuration

- **Reynolds-averaged simulation is performed within the following problem configuration:**



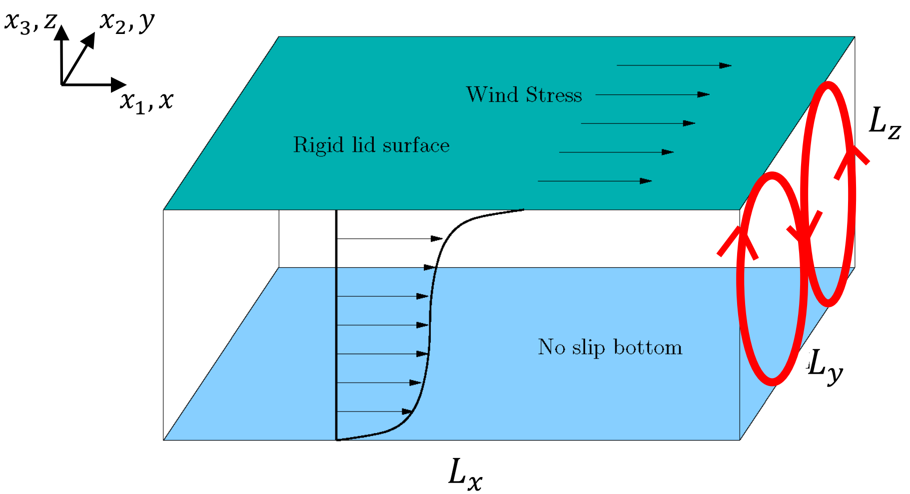
- Boundary conditions
  - Top is a wind-sheared rid lid
  - No-slip bottom
  - Periodicity in x and y

- For this configuration, formulation resolves
  - wind-driven mean current
  - largest LC (i.e. full-depth LC)

- Following the Gargett et al. field measurements of full-depth LC:
  - $L_z = 15\ m$
  - $L_y = 62.8\ m$ to resolve a pair of full-depth Langmuir cells

Problem configuration

- **Reynolds-averaged simulation is performed within the following problem configuration:**



- Uniform mesh
  - 64 elements in y
  - 64 elements in z
  - 4 elements in x (making problem 2D)

- Reynolds-averaged solution will be compared with LES performed on 3D mesh: 32x64x96

Reynolds-averaged formulation

$\langle u_i \rangle$ contains wind-driven mean current + full-depth LC

u_i' contains scales smaller than full-depth LC

continuity: $$\frac{\partial \langle u_i \rangle}{\partial x_i} = 0$$

Craik-Leibovich vortex force

momentum: $$\frac{\partial \langle u_i \rangle}{\partial t} + \langle u_j \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \langle p \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle u_i \rangle}{\partial x_j^2} - \frac{\partial \langle u_i' u_j' \rangle}{\partial x_j} + \epsilon_{ijk} U_i^S \langle \omega_k \rangle$$

Stokes drift velocity aligned with wind: $$U_1^S = \omega k a^2 \frac{\cosh(2kx_3)}{2\sinh^2(kL_z)} \qquad U_2^S = U_3^S = 0$$

following Gargett et al. field measurements of full-depth LC

Wind and wave forcing parameters:

Wind stress = 0.1 N/m^2

Wave height = 1.2 m
Wave period = 8 s
Wavelength = 90 m

Reynolds stress: $-\langle u_i' u_j' \rangle$

Reynolds-averaged formulation

Reynolds stress turbulence model:

$$-\langle u_i' u_j' \rangle = \nu_t \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) + \nu_t \left(\frac{dU_i^S}{dx_j} + \frac{dU_j^S}{dx_i} \right)$$

***K*-*ε* model:**

$$\nu_t = C_\mu \frac{k^2}{\varepsilon}$$

(eddy viscosity)

$$\frac{\partial k}{\partial t} + \langle u_j \rangle \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \varepsilon$$

(turbulent kinetic energy, TKE, (k) transport equation)

$$\frac{\partial \varepsilon}{\partial t} + \langle u_j \rangle \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} G_k + C_{2\varepsilon} \frac{\varepsilon^2}{k}$$

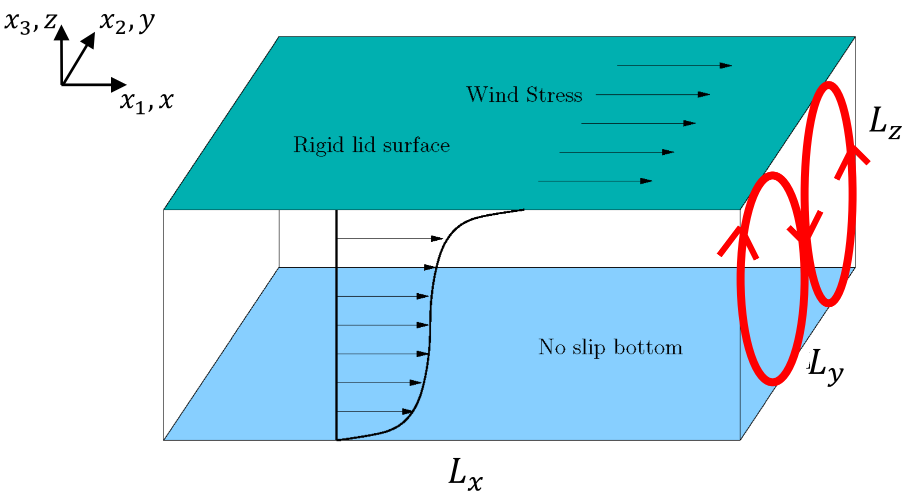
(TKE dissipation rate (ε) transport equation)

$$G_k = -\langle u_i' u_j' \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} - \langle u_1' u_3' \rangle \frac{dU_1^S}{dx_3}$$

(rate of production of TKE by mean Eulerian shear and Stokes drift shear)

Problem configuration

- **Initial development is performed within the following problem configuration:**

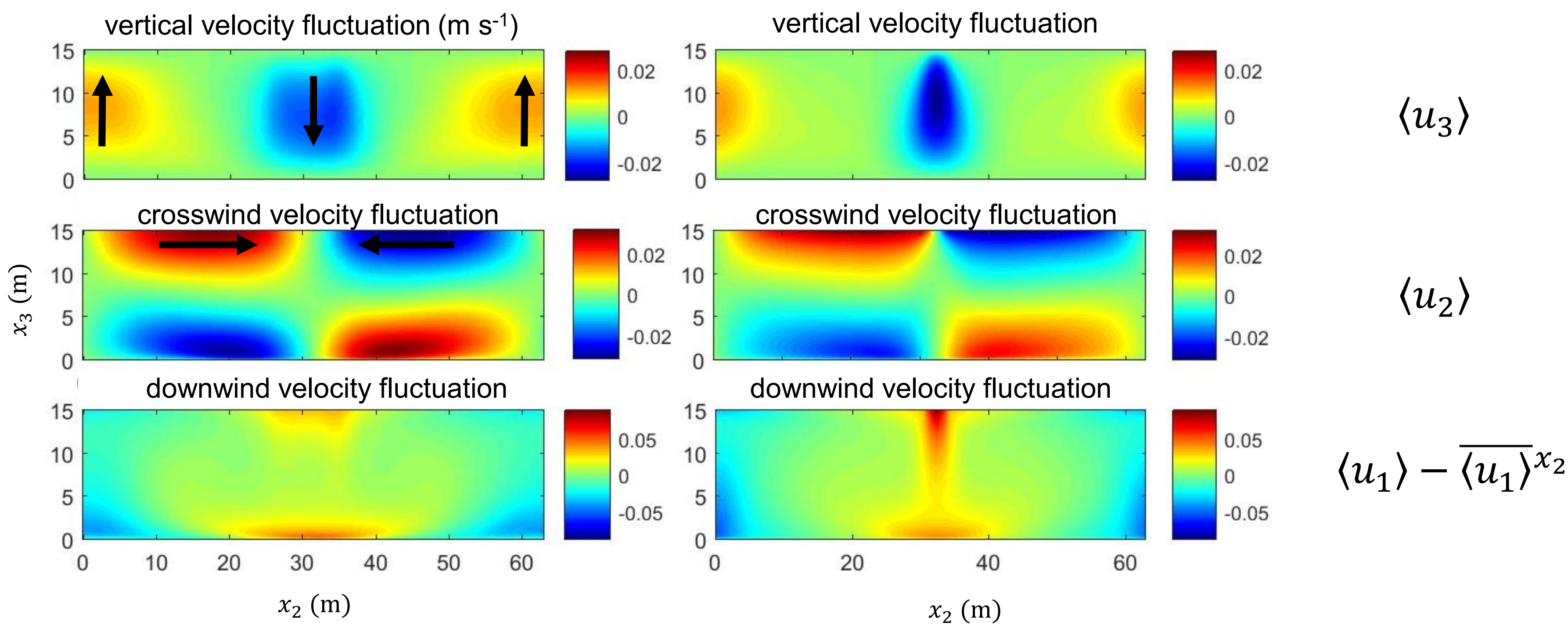


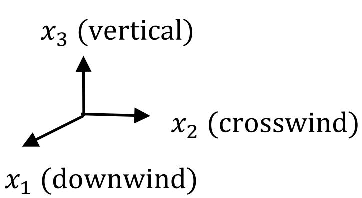
- Uniform mesh
  - 64 elements in y
  - 64 elements in z
  - 4 elements in x (making problem 2D)

- Reynolds-averaged solution with be compared with LES performed on 3D mesh: 32x64x96

Velocity fluctuations characterizing full-depth LC

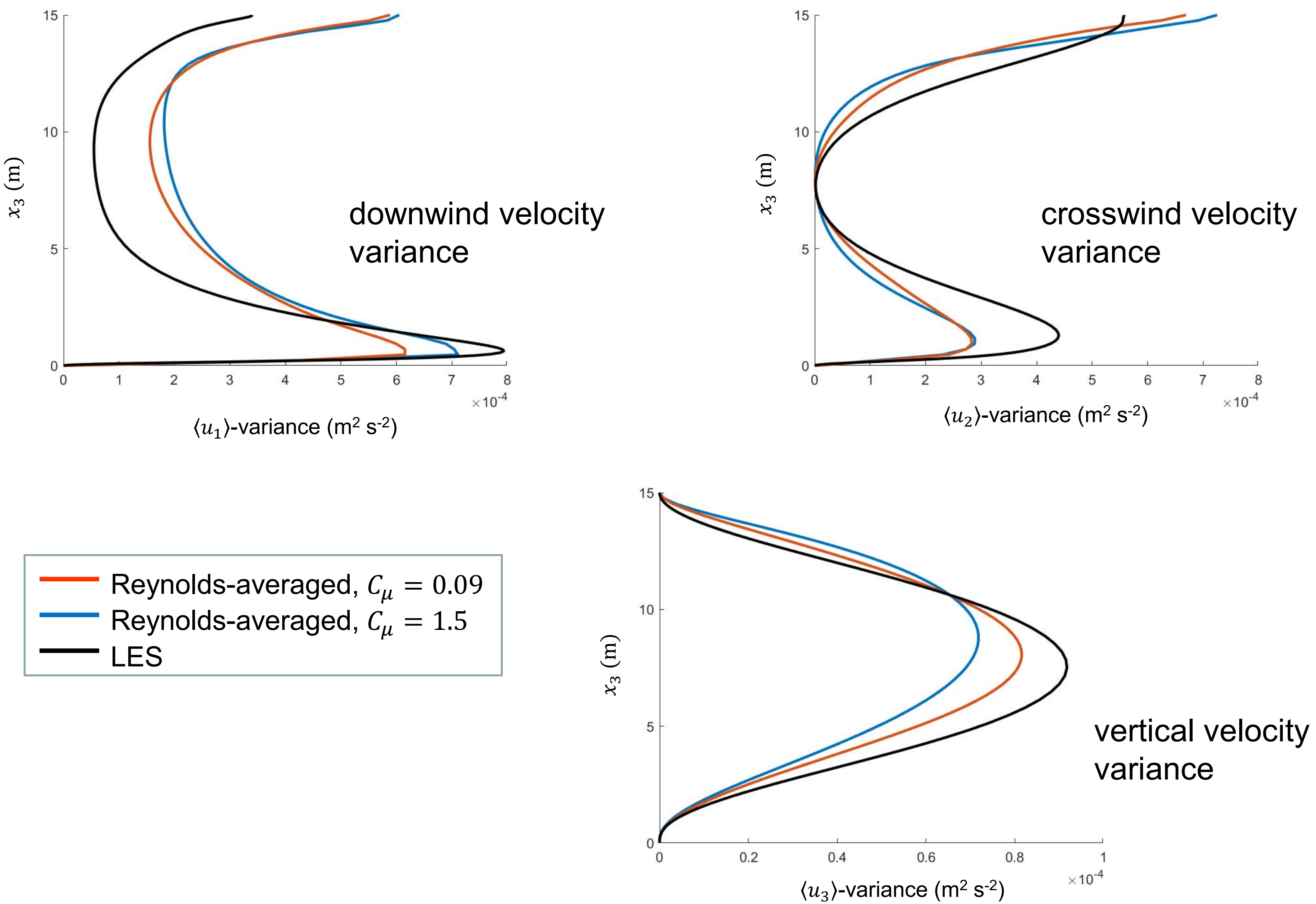
time-averaged LES   Reynolds-averaged



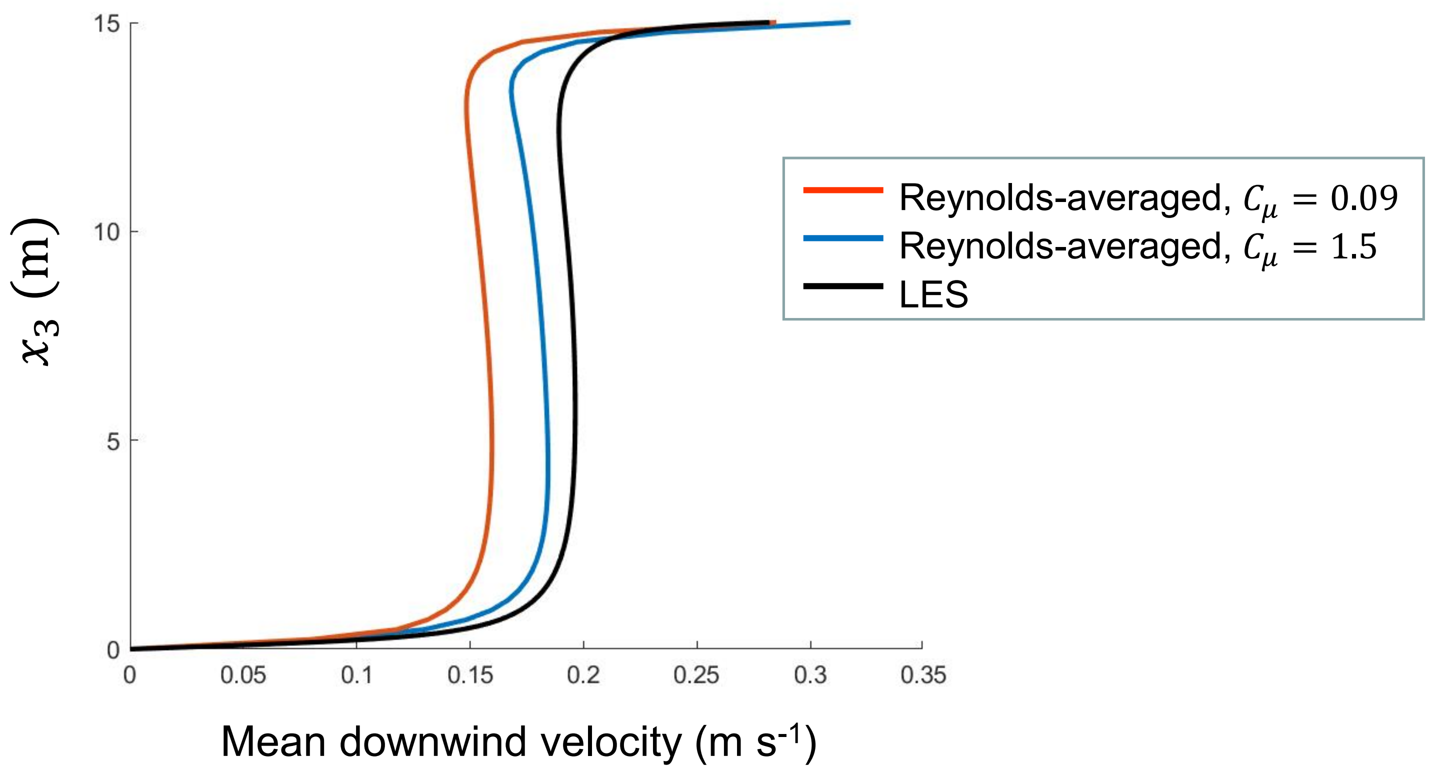


- **Reynolds-averaged formulation captures overall structure and intensity of full-depth LC**
- **See Tejada-Martinez et al. (2009) provides detailed comparison between the LES and field measurements of LC**

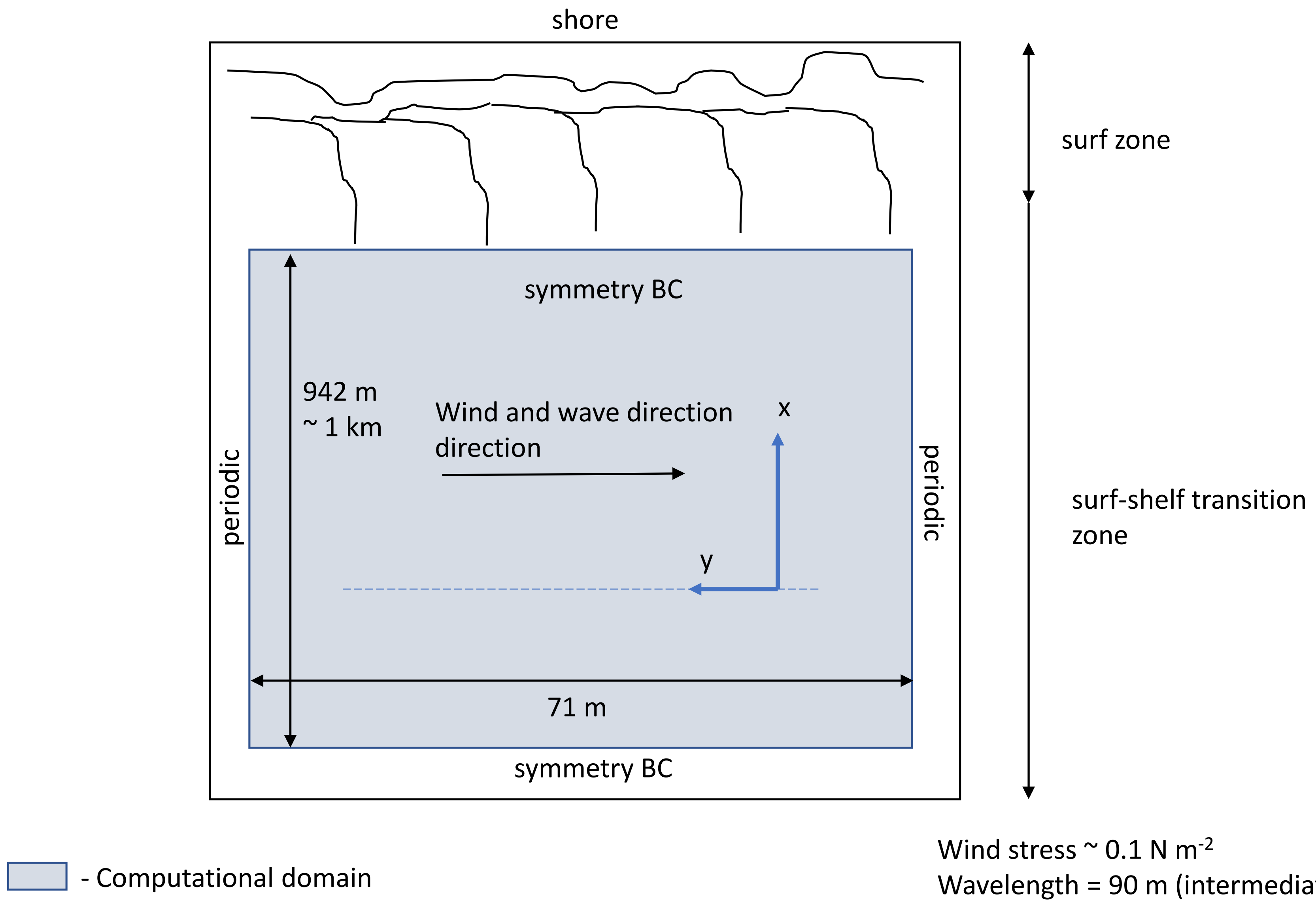
Velocity variance characterizing full-depth LC



Velocity variance characterizing full-depth LC

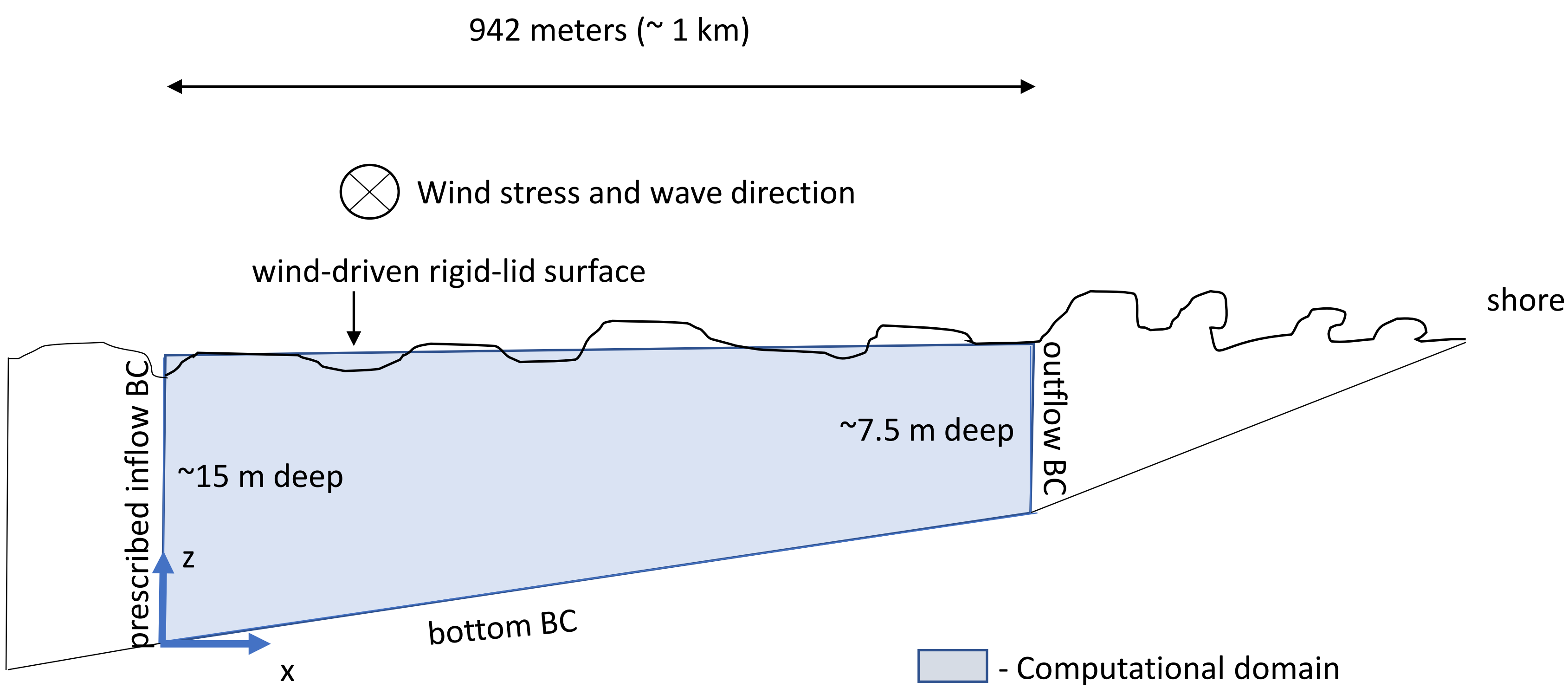


Reynolds-averaged simulation of Langmuir cells in variable depth shallow water



Wind stress ~ 0.1 N m^{-2}
Wavelength = 90 m (intermediate – shallow)
Wave height = 1.2 m

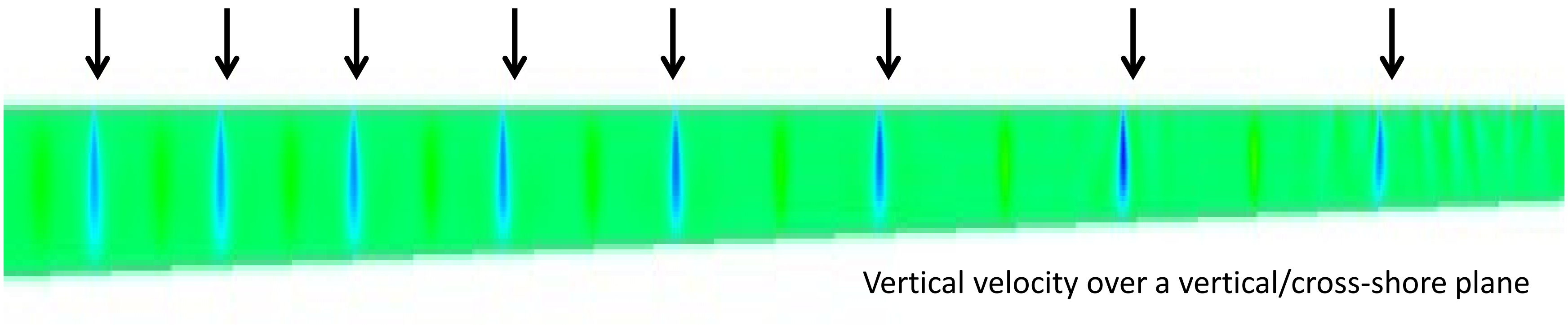
Reynolds-averaged simulation of Langmuir cells in variable depth shallow water



Wind stress = 0.1 N m^{-2}
Wavelength = 90 m (intermediate - shallow)
Wave height = 1.2 m

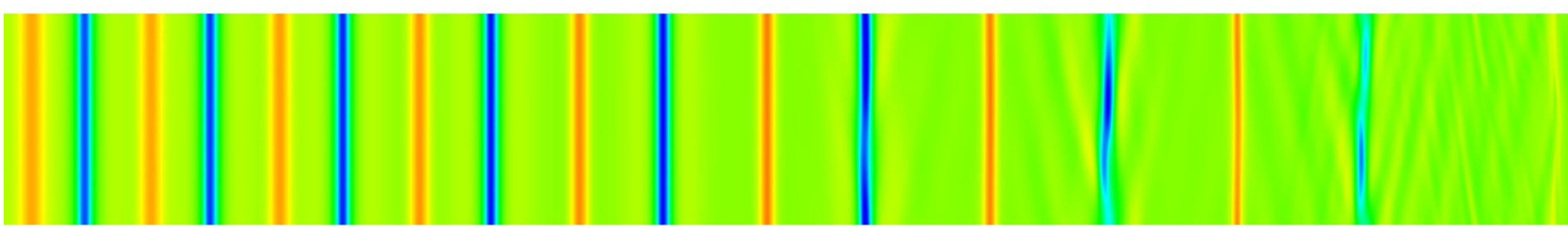
Reynolds-averaged simulation of Langmuir cells in variable depth shallow water

⊗ Wind stress and wave direction



Vertical velocity over a vertical/cross-shore plane

Wind and wave direction



Vertical velocity over horizontal plane at mid-depth

Conclusions

- A Reynolds-averaged formulation was introduced resolving the mean flow plus the largest scale of the Langmuir turbulence

- The largest scale consisted of full-depth Langmuir cells in the case studied here

- The formulation accounts for unresolved Langmuir turbulence through

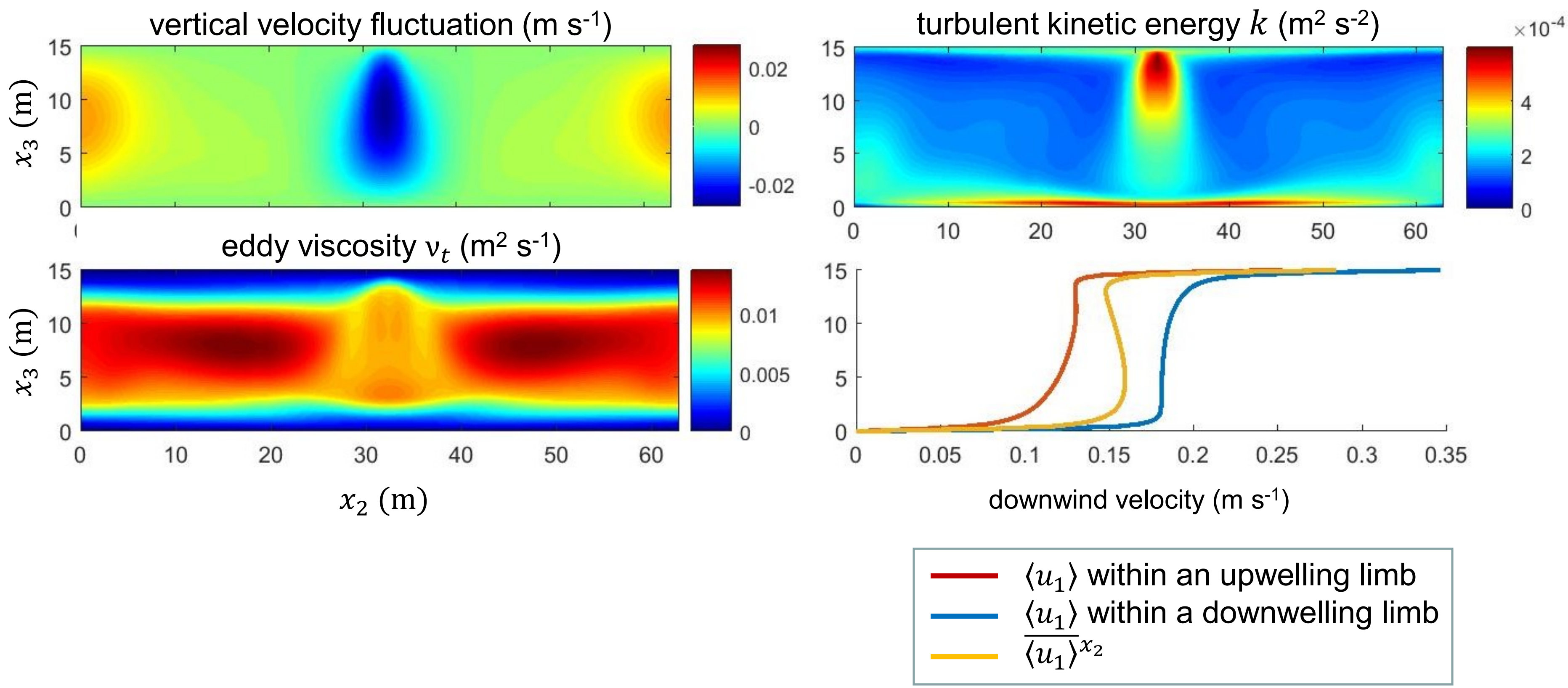
  (1) inclusion of production of TKE and TKE dissipation rate by Stokes drift shear
  (2) addition of Stokes shear to the Eulerian shear in the Reynolds stress closure accounting for non-local mixing induced by the Langmuir cells

- The formulation was found to give comparable results with LES in terms of structure and intensity of full-depth Langmuir cells resolved.

- Formulation was used to understand behavior of full-depth Langmuir cells in shallowing water. As depth becomes shallower, the width-to-depth ration of the cells becomes wider.

- In the future, this formulation will be used to understand the behavior of full-depth Langmuir cells in the presence of tidal-driven currents, density-driven currents and lateral boundaries.

Turbulent kinetic energy and eddy viscosty



Turbulent kinetic energy and eddy viscosty